Multi-objective optimization of HVAC system with an evolutionary computation algorithm

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Abstract

A data-mining approach for the optimization of a HVAC (heating, ventilation, and air conditioning) system is presented. A predictive model of the HVAC system is derived by data-mining algorithms, using a dataset collected from an experiment conducted at a research facility. To minimize the energy while maintaining the corresponding IAQ (indoor air quality) within a user-defined range, a multi-objective optimization model is developed. The solutions of this model are set points of the control system derived with an evolutionary computation algorithm. The controllable input variables — supply air temperature and supply air duct static pressure set points — are generated to reduce the energy use. The results produced by the evolutionary computation algorithm show that the control strategy saves energy by optimizing operations of an HVAC system.

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1. Introduction

Reducing the energy consumption of HVAC (heating, ventilating and air conditioning) systems is essential, as it constitutes over 50% of the building energy consumed in the US [1]. The operation of HVAC system is a critical activity in terms of optimizing the control settings to reduce the energy consumption, improving the system efficiency, and preserving the thermal comfort for the occupants. The performance of the existing HVAC system can be largely improved by adjusting the control set points to maximize the overall system capacity and efficiency.

This has intensified the research in modeling and optimization of energy use by HVAC systems. Analytical approaches [2] and simulation-based methods [3] were studied to model thermal behavior of buildings. Different control strategies, including experimental determination and intelligent control were applied [4, 5]. Zheng and Zaheer-Uddin [6] formulated the thermal process in a VAV (variable air volume) box with constraints on zone humidity. This provided daily operating strategies achieving optimal outdoor air-flow rates and energy savings. Wang and Jin [7] proposed a control strategy based on a genetic algorithm to search for the optimal settings of multiple variable processes. Differing from the data-driven models, the optimization algorithm was applied on the incremental models, with self-tuning ensuring the accuracy of the models. Fong et al. [8] discussed energy reduction by using an evolutionary programming approach to suggest optimal settings in response to the dynamic cooling loads and changing weather conditions. Ke and Mumma [9] studied the impact of tuning the supply air temperature set point in a VAV on energy consumption. Based on the steady-state models, Nassif et al. [10, 11] applied evolutionary algorithms to one- and two-objective optimization of an HVAC system, and the supervisory control strategies resulted in energy savings. Mossolly et al. [12] examined three control strategies developed by a genetic algorithm. Significant energy saving were achieved by varying system parameters. Kusiak and Li [13] applied an evolutionary strategy algorithm to solve a bi-objective optimization model to minimize the cooling output while maintaining the corresponding thermal properties.

An HVAC system is a complex, nonlinear, discrete system containing numerous variables and constraints. Therefore, the modeling and optimization of an HVAC system is a challenge for traditional mathematical models [14] and simulation approaches [15]. In this study, several data-mining algorithms are applied to build the predictive models. A multi-objective evolutionary computation algorithm is proposed for generating optimal control strategies of an existing HVAC system. Extensive test runs have been performed to determine the accuracy of the models. The optimized set points are used to minimize the energy consumption by maximizing the HVAC system efficiency. Different control strategies are given by weighting the objective functions to satisfy...
the preference of management operations. The energy use required for actual operations, along with the predictive results obtained from the trained and validated models, is compared to the one attained through the optimization controller set points. The resulting energy savings are presented.

2. HVAC system structure and description

The HVAC system being investigated is operated by the ERS (Energy Resource Station) in Ankeny, Iowa. It consists of two independent AHUs (air handling units) with the same zone loads and outside weather conditions. Each AHU serving four different thermal zones is set as a test area. Fig. 1 provides a schematic diagram of the existing AHU. For each thermal zone, a VAV box is connected to the AHU to maintain the comfort temperature of the thermal zone. The structure of the VAV box is shown in Fig. 2.

The experiment conducted in ERS was designed to investigate the impacts of AHU set points on the total energy consumption, since the HVAC system consumes a majority of the energy in a particular office building. Two set points, namely, AHU supply air temperature set point and static pressure set point, were adjusted for both air handling units, AHU-A and AHU-B. The supply air temperature set point varied from 52 °F (11.11 °C) to 63 °F (17.22 °C) with 1 °F (0.56 °C) increments. The supply air static pressure varied from 1.2 WG (0.3 kPa) to 1.8 WG (0.45 kPa) with 0.2 WG (0.05 kPa) increments. To simulate the impact of people and the lighting in the thermal zones, the internal load was divided into four stages reflecting the different thermal states at different time. The purpose of this experiment was to find the optimal set points minimizing the energy consumption while maintaining an acceptable level of IAQ.

The total energy consumed by the HVAC system includes two parts: the AHU and the VAV box. For the energy consumed in the AHU, three major categories, namely heat energy, fan energy, and pump energy, account for the total energy consumption. Since all three categories come from electricity, they can be calibrated by the meters originally installed in the system.

In the VAV box, the reheat load accounts for the maximum consumed energy. The VAV box supplies the conditioned air for a specific thermal zone to meet the comfort temperature of the zone envelope. By tuning the valve position and the dampers in the VAV box, the hot water flows through the coils adjusting to the actual requirements of the zone comfort. The reheat load is computed from equation (1) [16].

$$Q_{\text{Reheat}} = cm(T_{\text{VAV,AT}} - T_{\text{VAV,DT}})$$  \hspace{1cm} (1)

where $c$ is the heat capacity of the hot water, $m$ is the mass flow rate of the hot water, and $T_{\text{VAV,AT}}$ and $T_{\text{VAV,DT}}$ represent the entering and leaving water temperatures of the hydronic reheat coil, respectively.

The total energy consumed by the system is expressed in equation (2).

$$E_{\text{Total}} = E_{\text{Heat}} + E_{\text{Fan}} + E_{\text{Pump}} + Q_{\text{Reheat}}$$  \hspace{1cm} (2)

where $E_{\text{Heat}}$ is the energy consumed by the heat coil, $E_{\text{Fan}}$ is the energy consumed by the fan, $E_{\text{Pump}}$ represents for the energy use of water pump, and $Q_{\text{Reheat}}$ is the energy consumed during the reheat process in the VAV box.

The inherent nonlinearity and complexity of a typical HVAC system is difficult to accurately represent by a mathematical or a physics-based model. However, it can be easily captured by the empirical models developed from the process data. The data-driven models often outperform the traditional models of dynamic processes and prediction of performance metrics [17–19]. In this study, different data-mining algorithms are applied to derive the temporal process models.

To minimize the total energy $E_{\text{Total}}$, the function $y_{\text{Energy}}(t) = f(x_1, x_2)$ should be established between the output $E_{\text{Total}}$ and the input variables of the HVAC system. The function $f(\cdot)$ represents the AHU process, $x_1$ is a vector of $m$ controllable variables, $x_2$ is a vector of $n$ uncontrollable variables, and $y$ is the output variable. Both controllable and uncontrollable variables are used to represent the underlying dynamic process. To maintain the desired IAQ level, both room humidity and room comfort temperature are considered.
Let \( y_{\text{Energy}}(t) = f(x_1, x_2) \) be an objective function reflecting the energy consumption to be optimized with an evolutionary computation algorithm. The same parameters are used for modeling the indoor room temperature \( y_2(t) = f(x_1, x_2) \) and humidity \( y_3(t) = f(x_1, x_2) \), representing the IAQ at an acceptable level. The global optimal settings are achieved by minimizing the energy objective \( y_{\text{Energy}} \) while ensuring \( y_2(t) \) and \( y_3(t) \) vary within their constraint range. The optimization model proposed in this research is presented next.

\[
\begin{align*}
\text{Min} & \quad y_{\text{Energy}}(t) \\
\text{subject to:} & \quad y_2(t) = f(x_1, x_2) \\
& \quad y_3(t) = f(x_1, x_2) \\
& \quad x_1 \in [1, 2] \\
& \quad y_j \in [1, 3]
\end{align*}
\]

An evolutionary computation algorithm has been modified to search for the optimal control settings of the HVAC system. The two controllable variables, the AHU supply air temperature and the supply air static pressure, are considered to vary in a restricted range meeting the requirements of HVAC system. The room temperature and humidity are treated as constraints with their values changing in a certain range, so as not to sacrifice the environmental comfort while the output is minimized to reduce the energy consumption.

3. HVAC system modeling

The model proposed in this study has been tested on the data collected from February 4 to 15, 2010 at ERS. More than 300 variables have been captured reflecting the different characteristics of the HVAC system. The frequency of the original dataset is 1 min. To reduce the error produced by time delay and system error, the original 1 min data has been aggregated to 1 h interval data by calculating the mean value. In total, there are 576 observations (2 variables, twelve days of 1 h interval data) recorded in this dataset. Because of the existence of system and sensor errors, some abnormal data exist and will affect the accuracy of the experiment. After the pre-processing of the whole data, the dataset with 500 observations was randomly sampled and roughly divided into a training set (85% of dataset) and a testing set (15% of dataset). The dataset description is presented in Table 1.

3.1. Parameter selection and data dimensionality reduction

Some of the parameters collected are irrelevant or redundant in the modeling process. Therefore, parameter selection is essential before building the predictive model. The presence of irrelevant or redundant parameters in data mining may mask primary patterns. Redundant parameters also duplicate much or all of the information contained in one or more other parameters, making the modeling task much more difficult than it should be. Eliminating parameters that are less important or related may improve the accuracy, scalability, and comprehensibility of the resulting models and decrease the dimensionality which may greatly reduce the complexity of a model [20].

For example, the mixed air temperature was important in this experiment for prediction, as it directly reflected how much energy should be consumed to reheat the air up to the comfort temperature. However, it had almost the same distribution of the supply air temperature, and thus it could be discarded as duplicate information.

Wrapper algorithms [21,22] that use induction learning as evaluation functions were applied to select the most important parameters. A wrapper algorithm searches the space of all the possible parameters and evaluates each subset of parameters by building a model on each subset. The subset of parameters providing highest prediction accuracy is selected. Considering high computational cost, three widely used search methods, namely genetic algorithm, greedy, and linear forward, were used in the wrapper approach. Four different classifiers, namely linear regression, pace regression, SVM (support vector machine) regression, and MLP (multi-layer perceptron), were also used.

Since a single wrapper algorithm might dominate the features in some aspects, a vote method that is more robust to select the appropriate parameters than a single method, was applied to balance the selection of parameters by combining five different wrapper evaluation functions (Table 2) with 10-fold cross-validation. For each wrapper evaluator, the number of each candidate selected within the 10-fold iterative cross-validation was summed up. Then the results of the five different wrapper evaluators were aggregated to determine the importance of each parameter by counting the total number of times selected.

The parameter getting the more votes indicates the larger impact on the output of the HVAC system. The results of the five different combinations of wrapper evaluators are presented in Table 3.

<table>
<thead>
<tr>
<th>Dataset Description</th>
<th>No. of instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Model training: A random sample of 85% of the data</td>
</tr>
<tr>
<td>2</td>
<td>Model test: The remaining 15% of the data</td>
</tr>
</tbody>
</table>

Based on both domain knowledge and the results of the wrapper algorithm, the top 12 variables were selected for the potential candidates to construct the energy consumption model. The correlation coefficient [23] matrix was then applied to the selected parameters to reduce the linearity relationships among these variables, since some parameters may contain similar information and produce the same impact on the outputs. The results of the correlation coefficient are presented in Table 4.

Table 1. Five different algorithms of the wrapper evaluator.

<table>
<thead>
<tr>
<th>Evaluator</th>
<th>Classifier</th>
<th>Search method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wrapper</td>
<td>Linear regression</td>
<td>Genetic</td>
</tr>
<tr>
<td></td>
<td>Pace regression</td>
<td>Linear forward</td>
</tr>
<tr>
<td></td>
<td>Linear regression</td>
<td>Greedy</td>
</tr>
<tr>
<td></td>
<td>SVM regression</td>
<td>Genetic</td>
</tr>
<tr>
<td></td>
<td>MLP</td>
<td>Genetic</td>
</tr>
</tbody>
</table>
impacted by the weather patterns. The two controllable parameters, the AHU supply air temperature and the supply air duct static pressure set points, directly account for the system energy consumption. They are optimized in the next section, and the control strategy to generate them is discussed.

3.2. Construction and validation of the energy consumption model

After parameter selection and dimensionality reduction, the energy consumption model of the HVAC system is expressed in equation (4).

\[ Y_{\text{Energy}}(t) = f(x_{\text{SAT, Spr}}, x_{\text{SASP, Spr}}, x_{\text{Load}(t)}, x_{\text{Load}(t-1)}, x_{\text{SA, Hum}}, x_{\text{SA, Horz}}, x_{\text{IR Radio}}, x_{\text{Bar, Pres}}, x_{\text{OA, Temp}}, x_{\text{SO, Beam}}) \]

(4)

where \( Y_{\text{Energy}}(t) \) is the energy to be optimized; \( x \) represents all the inputs of this predictive model.

Five data-mining algorithms were used to extract the mapping between inputs and outputs: Exhaustive General CHAID (Chi-square Automatic Interaction Detector) [24], Boosting Tree [26], Random Forest [27], SVM (Support Vector Machines) [28], and MLP [29].

The Exhaustive CHAID algorithm is derived from the standard CHAID [25], which is a type of decision tree allowing multiple splits of nodes and can be used for detection of interaction between variables in regression and classification analysis. It allows for more comprehensive merging than standard CHAID.

Boosting tree is a typical machine learning meta-algorithm for performing supervised learning. Boosting is an iterative procedure used to adaptively modify the distribution of training examples so that the base predictors will mostly concentrate on learning instances misclassified by the previous biased examples.

Random Forest is a class of ensemble methods consisting of multiple decision trees, where each tree is generated based on the values of an independent set of random variables. Unlike the adaptive approach used in Boosting, the random variables are generated from a fixed probability distribution.

SVM is a supervised learning algorithm based on kernel functions, and it is applied to binary classification and regression. Using specific kernel functions, the original vector space is transformed into a higher-dimensional space where a separated hyperplane is constructed with the maximum margin.

The MLP Ensemble method presents a combination of multiple models to leverage the strength of multiple MLP models in achieving better prediction accuracy than any individual model does.

The five different data-mining algorithms have been tested for the construction of predictive models. In order to evaluate the performance of different algorithms, the following four metrics (see equations (5)–(10)) have been used to measure the prediction accuracy of the model: the MAE (mean absolute error), the Std_AE (standard deviation of absolute error), the MAOE (mean absolute percentage error) and the Std_APE (standard deviation of absolute percentage error) [30]:

\[ AE = \frac{1}{n} \sum_{i=1}^{n} |\tilde{y} - y| \]

(5)

\[ MAE = \frac{\sum_{i=1}^{n} AE}{N} \]

(6)

| Table 3 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| No. of times selected | No. of times selected | No. of times selected | No. of times selected | No. of times selected | Total |
| Internal load (t - 1) | 10 | 10 | 10 | 10 | 50 |
| SA set point | 10 | 10 | 10 | 9 | 49 |
| MA-Temp | 10 | 10 | 10 | 9 | 49 |
| OA-Hum | 10 | 10 | 10 | 8 | 43 |
| SPSS set point | 10 | 10 | 10 | 3 | 9 |
| BAR-Pres | 10 | 10 | 7 | 10 | 39 |
| OA-Temp | 3 | 10 | 8 | 10 | 39 |
| SOL-Beam | 10 | 7 | 5 | 10 | 33 |
| SA-Hum | 9 | 9 | 4 | 9 | 33 |
| IR-Rad | 8 | 9 | 5 | 2 | 29 |
| SOL-Hor | 2 | 7 | 5 | 3 | 27 |
| WIND-Dir | 8 | 5 | 7 | 1 | 26 |
| WIND-Vel | 2 | 8 | 9 | 1 | 25 |
| OA-CO2 | 0 | 2 | 2 | 3 | 11 |

| Table 4 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Internal load (t) | Internal load (t - 1) | SA set point | SASP set point | MA-temp | SA-Hum | BAR-Pres | IR-Rad | OA-Hum | OA-Temp | SOL-beam | SOL-Hor |
| 1.00 | 0.45 | 0.01 | 0.02 | 0.02 | 0.02 | 0.06 | 0.06 | 0.07 | 0.05 | 0.01 | 0.08 |
| 0.45 | 1.00 | -0.02 | -0.02 | 0.00 | 0.00 | -0.06 | -0.06 | 0.00 | 0.00 | -0.05 | -0.04 | -0.12 |
| 0.01 | -0.02 | 1.00 | 0.00 | 0.00 | -0.06 | -0.06 | 0.00 | 0.00 | -0.05 | 0.00 | -0.04 | -0.12 |
| -0.02 | -0.01 | -0.02 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.13 | -0.18 | -0.29 |
| 0.02 | 0.00 | 0.00 | 0.99 | -0.06 | 1.00 | -0.54 | -0.71 | 0.04 | 0.15 | -0.18 | -0.28 | -0.03 |
| 0.02 | 0.00 | -0.54 | 0.00 | -0.54 | 1.00 | 0.13 | 0.65 | 0.32 | 0.80 | -0.09 | -0.10 |
| -0.06 | -0.06 | -0.72 | 0.03 | -0.71 | 0.13 | 1.00 | -0.19 | -0.53 | 0.06 | 0.38 | -0.12 |
| 0.06 | -0.02 | 0.04 | 0.00 | 0.04 | 0.65 | -0.19 | 1.00 | 0.38 | 0.77 | -0.34 | -0.06 |
| -0.07 | -0.05 | 0.13 | -0.35 | 0.15 | 0.32 | 0.53 | 0.38 | 1.00 | 0.10 | -0.47 | -0.46 |
| 0.05 | 0.00 | -0.18 | 0.13 | -0.18 | 0.80 | 0.06 | 0.77 | 0.10 | 1.00 | -0.05 | 0.09 |
| 0.01 | -0.04 | -0.29 | 0.20 | -0.28 | -0.09 | 0.38 | -0.34 | -0.47 | -0.05 | 1.00 | 0.51 |
| 0.08 | -0.12 | 0.00 | 0.57 | -0.03 | -0.10 | 0.12 | -0.06 | -0.46 | 0.09 | 0.51 | 1.00 |

can be described as follows: the same methodology introduced in Section 3.2, the IAQ model is used to indoor air quality while minimizing the energy consumption. Using temperature and humidity should be constructed so as to conform to the implementation of changeable settings. Models of room temperature and the supply air static pressure set point change, below 30% to ensure the thermal comfort. As the supply air temperature and the supply air static pressure set point change, the room temperature and humidity are also affected by the supply air temperature set point change, the previous state of system internal load, supply air humidity % RH, solar normal flux B/HF2, infrared radiation Barbarenometric pressure mbar, outside air temperature °C, solar beam intensity B/HF2.

\[ APE = \frac{y - \hat{y}}{y} \]  

(7)

\[ MAPE = \frac{\sum_{i=1}^{n} APE_i}{N} \]  

(8)

\[ \text{Std}_{\text{AE}} = \sqrt{\frac{\sum_{i=1}^{n} (AE_i - MAE)^2}{N-1}} \]  

(9)

\[ \text{Std}_{\text{MAPE}} = \sqrt{\frac{\sum_{i=1}^{n} (APE_i - MAPE)^2}{N-1}} \]  

(10)

where AE in equation (2) represents the absolute error, \( \hat{y} \) is the predicted value obtained from the model, \( y \) is the actual target value measured, and \( N \) is the number of data points used for training or testing.

Table 6 presents the prediction performance of the energy consumption models built by five different-mining algorithms. The test results of the MLP neural network for this model are shown in Fig. 3. The chart shows that the predicted values closely track the observed ones at most of the points.

4. Optimization algorithm

4.1. Model formulation

The optimization process seeks the set point values minimizing the total energy consumption of the HVAC system. This is done by applying the evolutionary computation algorithm [31]: the set points were optimized to maximize the energy savings. The optimization model is formed through the identification of the problem parameters, the objective functions, and the constraints.

4.1.1. Model parameters

The model parameters of the HVAC system have been determined by the wrapper algorithm described in Section 3.1. Table 5 lists each parameter used for the optimization process. The two controllable parameters, the AHU supply air temperature and the supply air static pressure set points, are to be varied to obtain the optimal solutions. As uncontrollable input parameters are essentially independent of the controllable ones, the values of uncontrollable variables, such as supply air humidity, outside air temperature, and supply air static pressure set point change, below 30% to ensure the thermal comfort. As the supply air temperature and the supply air static pressure set point change, the room temperature and humidity are also affected by the supply air temperature set point change, the previous state of system internal load, supply air humidity % RH, solar normal flux B/HF2, infrared radiation Barbarenometric pressure mbar, outside air temperature °C, solar beam intensity B/HF2.

\[ y_{\text{Humidity}}(t) = f(x_{\text{SAT,Spt}} \cdot x_{\text{SASP,Spt}} \cdot x_{\text{Load}(t)} \cdot x_{\text{Load}(t-1)} \cdot x_{\text{SA,Humid}} \cdot x_{\text{SOL,Horz}} \cdot x_{\text{IR,Radia}} \cdot x_{\text{Bar,Pres}} \cdot x_{\text{OA,TEMP}} \cdot x_{\text{SOL,Beam}}) \]  

(12)

where \( y_{\text{Temp}}(t) \) and \( y_{\text{Humidity}}(t) \) are constraints to meet the IAQ of HVAC system; \( x \) represents all the inputs of this predictive model.

The MLP algorithm and the same dataset are used to construct the two models. The test results can be seen in Fig. 4 and 5.

3.3. Construction and validation of the IAQ models

In this HVAC system examined, the value of room temperature is maintained from 69°F to 73°F and the room humidity is controlled below 30% to ensure the thermal comfort. As the supply air temperature and the supply air static pressure set points change, the room temperature and humidity are also affected by the implementation of changeable settings. Models of room temperature and humidity should be constructed so as to conform to the indoor air quality while minimizing the energy consumption. Using the same methodology introduced in Section 3.2, the IAQ model can be described as follows:

\[ y_{\text{Temp}}(t) = f(x_{\text{SAT,Spt}} \cdot x_{\text{SASP,Spt}} \cdot x_{\text{Load}(t)} \cdot x_{\text{Load}(t-1)} \cdot x_{\text{SA,Humid}} \cdot x_{\text{SOL,Horz}} \cdot x_{\text{IR,Radia}} \cdot x_{\text{Bar,Pres}} \cdot x_{\text{OA,TEMP}} \cdot x_{\text{SOL,Beam}}) \]  

(11)

Fig. 3. Test results produced by the model.
temperature and other outside weather patterns, can be fixed in seeking the optimal control settings at each time stamp.

4.1.2. Objective functions

The optimal control strategy determines two set points to minimize the energy use. The total energy consumption, including the fan, pump, and reheat power, is computed from equation (2). The input–output relationship was expressed by the HVAC system model presented above in equations (3), (11) and (12). The energy objective function is to be minimized while the other two — temperature and humidity objective functions — are treated as constraints to satisfy the indoor air quality of the AHU system.

4.1.3. Constraints

The constraints in the model are due to the upper and lower limits imposed on the parameters of the HVAC system and the IAQ models. The value of the supply air temperature set point, the supply air duct static pressure set point, indoor room temperature, and room humidity are restricted within the limits:

- Supply air temperature must vary between 52 °F (11.11 °C) and 63 °F (17.22 °C).
- Supply air duct static pressure must vary between 1.2 WG (0.3 kPa) and 1.8 WG (0.45 kPa).
- Room temperature must be maintained between 70 °F (21.11 °C) and 72 °F (22.22 °C).
- Room humidity must be controlled below 30%.

Consequently, the optimization model (13) is expressed as minimizing the objective function (4) with control parameters varying within their bounds.

\[
\text{min} \left( y_{\text{Energy}}(t) \right) \quad \text{subject to :}
\]

\[
y_{\text{Energy}}(t) = \begin{pmatrix} X_{\text{SAT Spt}}, X_{\text{SASP Spt}}, X_{\text{Load(t)}}, X_{\text{Load(t-1)}}, X_{\text{SA_Humd}}, X_{\text{SOL_Horz}}, X_{\text{IR_Radia}}, X_{\text{Bar_Pres}}, X_{\text{OA_TEMP}}, X_{\text{SOL_Beam}} \end{pmatrix}
\]

\[
y_{\text{Temp}}(t) = \begin{pmatrix} 11.11 \leq X_{\text{SAT Spt}} \leq 17.22, \\ 0.3 \leq X_{\text{SASP Spt}} \leq 0.45, \\ 20.56 \leq y_{\text{Temp}}(t) \leq 22.78, \\ 5 \leq y_{\text{Humid}}(t) \leq 25 \end{pmatrix}
\]

Solving such a nonlinear multi-objective optimization problem is a challenge. For the typical HVAC system, the procedure of computation is also complex and time-consuming, resulting in the difficulty of getting the results for some large datasets. In order to simplify the problem and reduce the calculation time, the two constraint functions \( y_{\text{Temp}}(t) \) and \( y_{\text{Humidity}}(t) \) are assigned into one-objective function shown in equation (14):

\[
y_{\text{Constraints}}(t) = \max \left\{ 0, 20.56 - y_{\text{Temp}}(t) \right\} + \max \left\{ 0, y_{\text{Temp}}(t) - 22.78 \right\} + \max \left\{ 0, 5 - y_{\text{Humidity}}(t) \right\} + \max \left\{ 0, y_{\text{Humidity}}(t) - 25 \right\}
\]

\[ (14) \]

Fig. 4. Test results for the room humidity model.

Fig. 5. Test results for the room temperature model.

Fig. 6. Feasible solutions obtained after 150 generations at some time stamp.
As the constraints are satisfied, each of the four terms in equation (14) will remain 0 and the sum equals 0. However, since room humidity and temperature are not at the same scale, the constraint function may not accurately reflect the influences of them. It is possible room humidity may dominate the result because of its high value. The values of room humidity and temperature are then normalized to eliminate the deviation. The revised function is shown in equation (15).

\[
\begin{align*}
\min \left( y_{\text{Energy}}(t) \right) \\
\text{subject to:} \\
y_{\text{Energy}}(t) &= f(x_{\text{SAT,Spt}}, x_{\text{SASP,Spt}}, x_{\text{Load(t-1)}}, x_{\text{CHWC, LVL}}, x_{\text{SA_Humid}}, x_{\text{SA_Horiz}}, x_{\text{OA_Humid}}, x_{\text{OA_TEMP}}) \\
y_{\text{Constraints}}(t) &= w_1 \left( \max \left\{ 0, 0.2056 - y_{\text{Temp}}(t) \right\} + \max \left\{ 0, y_{\text{Temp}}(t) - 22.78 \right\} \right) \\
&+ w_2 \left( \max \left\{ 0.5 - y_{\text{Humidity}}(t) \right\} + \max \left\{ 0, y_{\text{Humidity}}(t) - 25 \right\} \right) \\
&\geq 11.11 - x_{\text{SAT,Spt}} \leq 17.22 \\
&0.3 \leq x_{\text{SASP,Spt}} \leq 0.45
\end{align*}
\]

(15)

4.2. Evolutionary computation algorithm

In this study, a modified SPEA-LS (Strength Pareto Evolutionary Algorithm with Local Search) is used to solve the model (12). The algorithm is presented below:

Step 1: Initialize a population \( P \) and create an empty external population \( P^* \) to store elite solutions.

Step 2: Find non-dominated solutions in \( P \) and copy them into \( P^* \).

Step 3: Find non-dominated solutions in \( P \) and update the elite population \( P^* \).

Table 7
Description of the eleven weight assignment scenarios.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Weight assigned to energy</th>
<th>Weight assigned to IAQ constraints</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( w_1 = 1.0 )</td>
<td>( w_2 = 0 )</td>
<td>No IAQ constraints</td>
</tr>
<tr>
<td>2</td>
<td>( w_1 = 0.9 )</td>
<td>( w_2 = 0.1 )</td>
<td>Preference to energy saving</td>
</tr>
<tr>
<td>3</td>
<td>( w_1 = 0.8 )</td>
<td>( w_2 = 0.2 )</td>
<td>Preference to energy saving</td>
</tr>
<tr>
<td>4</td>
<td>( w_1 = 0.7 )</td>
<td>( w_2 = 0.3 )</td>
<td>Preference to energy saving</td>
</tr>
<tr>
<td>5</td>
<td>( w_1 = 0.6 )</td>
<td>( w_2 = 0.4 )</td>
<td>Preference to energy saving</td>
</tr>
<tr>
<td>6</td>
<td>( w_1 = 0.5 )</td>
<td>( w_2 = 0.5 )</td>
<td>Equal importance to both objects</td>
</tr>
<tr>
<td>7</td>
<td>( w_1 = 0.4 )</td>
<td>( w_2 = 0.6 )</td>
<td>Preference to IAQ maintenance</td>
</tr>
<tr>
<td>8</td>
<td>( w_1 = 0.3 )</td>
<td>( w_2 = 0.7 )</td>
<td>Preference to IAQ maintenance</td>
</tr>
<tr>
<td>9</td>
<td>( w_1 = 0.2 )</td>
<td>( w_2 = 0.8 )</td>
<td>Preference to IAQ maintenance</td>
</tr>
<tr>
<td>10</td>
<td>( w_1 = 0.1 )</td>
<td>( w_2 = 0.9 )</td>
<td>Preference to IAQ maintenance</td>
</tr>
<tr>
<td>11</td>
<td>( w_1 = 0 )</td>
<td>( w_2 = 1.0 )</td>
<td>Maximization of thermal comfort</td>
</tr>
</tbody>
</table>

In the practical operation of HVAC system, it is possible that some constraint is of more importance or at least over than some other constraint. In different time period of the year, managers may emphasize on different objectives. For example room temperature should be paid more attention to room humidity in winter since the air humidity is really low and is not a big issue. Taking the effects of different constraints, weights should be assigned to each constraint to meet the specific preference of HVAC system. The final constraint functions are presented in eqs. (16) and (17):

\[
y_{\text{Constraints}}(t) = \begin{cases} 
  y_{\text{Energy}}(t) & \\
  y_{\text{Constraints}}(t) & \\
  \end{cases}
\]

\[
\begin{align*}
\begin{align*}
\min \left( y_{\text{Energy}}(t) \right) \\
\text{subject to:} \\
y_{\text{Energy}}(t) &= f(x_{\text{SAT,Spt}}, x_{\text{SASP,Spt}}, x_{\text{Load(t-1)}}, x_{\text{CHWC, LVL}}, x_{\text{SA_Humid}}, x_{\text{SA_Horiz}}, x_{\text{OA_Humid}}, x_{\text{OA_TEMP}}) \\
y_{\text{Constraints}}(t) &= w_1 \left( \max \left\{ 0, 0.2056 - y_{\text{Temp}}(t) \right\} + \max \left\{ 0, y_{\text{Temp}}(t) - 22.78 \right\} \right) \\
&+ w_2 \left( \max \left\{ 0.5 - y_{\text{Humidity}}(t) \right\} + \max \left\{ 0, y_{\text{Humidity}}(t) - 25 \right\} \right) \\
&\geq 11.11 - x_{\text{SAT,Spt}} \leq 17.22 \\
&0.3 \leq x_{\text{SASP,Spt}} \leq 0.45
\end{align*}
\end{align*}
\]

(16)

(17)

Table 8
Weight assignment of 11 scenarios.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Total energy</th>
<th>Recommended SA setting</th>
<th>Recommended SASP setting</th>
<th>Temperature</th>
<th>Humidity</th>
<th>IAQ violation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26061.89</td>
<td>14.76</td>
<td>0.45</td>
<td>20.63</td>
<td>22.90</td>
<td>0.26</td>
</tr>
<tr>
<td>2</td>
<td>26063.17</td>
<td>14.97</td>
<td>0.45</td>
<td>20.67</td>
<td>23.14</td>
<td>0.28</td>
</tr>
<tr>
<td>3</td>
<td>26065.49</td>
<td>15.03</td>
<td>0.45</td>
<td>20.69</td>
<td>23.21</td>
<td>0.23</td>
</tr>
<tr>
<td>4</td>
<td>26076.04</td>
<td>14.97</td>
<td>0.45</td>
<td>20.72</td>
<td>23.39</td>
<td>0.21</td>
</tr>
<tr>
<td>5</td>
<td>26073.45</td>
<td>15.42</td>
<td>0.45</td>
<td>20.78</td>
<td>23.66</td>
<td>0.18</td>
</tr>
<tr>
<td>6</td>
<td>26185.11</td>
<td>15.84</td>
<td>0.45</td>
<td>20.90</td>
<td>24.13</td>
<td>0.11</td>
</tr>
<tr>
<td>7</td>
<td>26249.09</td>
<td>16.07</td>
<td>0.45</td>
<td>20.97</td>
<td>24.39</td>
<td>0.07</td>
</tr>
<tr>
<td>8</td>
<td>26450.17</td>
<td>16.53</td>
<td>0.43</td>
<td>21.10</td>
<td>25.11</td>
<td>0.01</td>
</tr>
<tr>
<td>9</td>
<td>26450.17</td>
<td>16.53</td>
<td>0.43</td>
<td>21.10</td>
<td>25.11</td>
<td>0.01</td>
</tr>
<tr>
<td>10</td>
<td>26450.17</td>
<td>16.53</td>
<td>0.43</td>
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<td>11</td>
<td>26450.17</td>
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<td>0.43</td>
<td>21.10</td>
<td>25.11</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Step 1: Specify an initial solution
Step 2: Examine n neighborhood solutions \( y = \{x_1, x_2, \ldots, x_n\} \) of the current solution \( x \).
Step 3: Evaluate all the neighborhood solutions in \( y \) and find the best one. If a solution which is better than \( x \) is found, replace the current solution \( x \) with the better one.
Step 4: Return to Step 1 until all the individuals are examined.

In the process of assigning the fitness for individuals \( P \) and \( P^* \), two different functions are applied to ensure the elite solution and retain the variety of the population. For individuals in elite population \( P^* \), the fitness function is:

\[
S_i = \frac{n_i}{N + 1}
\]

where \( S_i \) is the fitness value of \( i \)-th elite individual in non-dominated population \( P^* \), \( n_i \) is the number of individuals in \( P \) that solution \( i \) dominates, and \( N \) is the population size of \( P \).

For the individuals in current population \( P \), the fitness function is:

\[
S_j = 1 + \sum_{i \in F} S_i
\]

where \( S_i \) is the fitness of \( i \)-th individual in the current population \( P, F \) is the elite set from \( P^* \).

The control parameters of the evolutionary computation algorithm must be adjusted to provide the best performance, i.e., minimum energy consumption of the HVAC system with the least sacrifice of air quality. The mutation operator in Step 6 is realized by adding noise \( \Delta x_i \) (from a Gaussian distribution with zero mean and standard deviation \( \sigma \)) to the \( i \)-th controllable variable \( x_i \). The newly generated solution is \( \hat{x}_i = x_i + N(0, \sigma) \). In global search, the noise value for \( x_1(t) \) and \( x_2(t) \) is set to 0.5 and 0.05, respectively. In local search, the noise is set to 0.1 and 0.01, respectively. The population size is \( p_z = 100 \). The stopping criterion is set at 150 generations.

### 4.3. Optimization results and discussion

The proposed evolutionary computation algorithm was applied to solve the optimization model at each time stamp. Optimized control settings of the existing HVAC system, namely the supply air temperature and the supply air duct static pressure set points, were obtained. The performance of the HVAC system model has been validated. The bounds set for supply air temperature and supply air static pressure may be slightly violating the constraints during the generations.

Fig. 6 shows the values of the objective function of model (18) generations of the evolutionary computation algorithm. A decrease in constraints indicating the better air quality requires an increase in...
energy demand. When the constraints are satisfied (the left point on the boundary), the system consumes more energy than at other points. If the decision is to save more energy, the consequence must be the sacrifice of the thermal comfort and air quality. According to the different preferences of management, the results will largely differ if different weights are assigned to each objective function. A trade-off must be considered to meet the specific requirements of different systems. If we assign \( w_1 \) to objective function one which considers the energy, and \( w_2 \) for objective function two representing the violation of air quality, the different combination of \( w_1 \) and \( w_2 \) will give the different preferences. The optimal solution is selected from the final elite set by the weighted normalized objective function (17). The following two equations show the following process.

\[
\text{Objective} = w_1^* \frac{\text{Objective}_1 - \text{Objective}_{1\text{min}}}{\text{Objective}_{1\text{max}} - \text{Objective}_{1\text{min}}} + w_2^* \frac{\text{Objective}_2 - \text{Objective}_{2\text{min}}}{\text{Objective}_{2\text{max}} - \text{Objective}_{2\text{min}}}
\]

(21)

where \( w_1 \) and \( w_2 \) represent the user-defined weights indicating the importance of \( \text{Objective}_1 \) and \( \text{Objective}_2 \), respectively; \( \text{Objective}_1 \) represents the energy consumption and \( \text{Objective}_2 \) represents the violation of air quality. Note that \( w_1 + w_2 = 1 \).

Let \( w_1 \) varies from 1 to 0 with 0.1 decrement while \( w_2 \) varies from 0 to 1 with 0.1 increment. Eleven scenarios representing different assignments of weights to the objectives have been created in Table 7. Table 8 lists the optimal solutions for each of the 11 scenarios at some time stamp. After assigning different weights to the objectives, the bi-objective optimization model is transformed into a single objective model to be minimized with the objective function shown in (21). Scenario 1 has the lowest total energy consumption as the air quality is not considered. As the weight assigned to the IAQ increases, the total energy saving is reduced.

To compare the optimal solutions with the actual energy consumption, Scenario 11 is selected. Figs. 7 and 8 compare the original and recommended control settings of the supply air temperature and the static pressure.

Based on the optimal control settings, the corresponding total energy and the room air quality metrics are estimated using the energy and the IAQ models of Section 3. Figs. 9–11 compare the original and optimized energy as well as the IAQ indexes.

Fig. 9 presents that the total energy consumption is reduced after the recommended control settings are implemented. The corresponding values of IAQ indexes are presented in Figs. 10 and 11. At times, minimization of the energy is compromised in order to maintain the required IAQ indexes. The total energy saved for Scenario 11 is 21.4% compared to 22.6% Scenario 1 with no IAQ constraints included.

The results discussed above represent only the two extreme situations of weights assignments. Different trade-off can be found by changing the weights to the corresponding objective. Table 9 presents the final optimal solutions after assigning different weights to the objectives. As the larger weight is assigned to IAQ constraints, more energy is consumed while the air quality is improved. Fig. 12 shows the gradient trend of optimized energy use and IAQ violations.

Fig. 12 demonstrates that when the weight associated with the energy objective decreases from 0.6 to 0.4, the energy consumption increases and the adherence of IAQ indexes improves. Both, the energy consumption and IAQ indexes are minimally affected when the two weights change from 1 to 0.6 and 0 to 0.4, respectively. The solution zone when the values of the weights are close to each other is sensitive. By changing the weights, the model allows for incorporation of user’s preferences.

5. Conclusion

The proposed data-mining and optimization approach was applied to an existing HVAC system at the ERS. A predictive model of energy consumption, room temperature, and humidity were generated by a MLP algorithm and aggregated into a model optimizing energy consumption. The model constructed in the paper was solved with an evolutionary computation algorithm to produce optimized set points of the supply air temperature and the supply air static pressure of the HVAC system. The optimized set points determined by solving a model with two-objectives resulted in

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Fig. 10. The observed and optimized room temperature for Scenario 11.

Fig. 11. The observed and optimized room humidity for Scenario 11.

Fig. 12. The optimized energy consumed versus IAQ violation.
energy saving of 21.4% without violating the indoor air quality constraints. When occasional violations of air quality were allowed the energy savings increased to 22.6%.

The approach presented in the paper, is suitable for implementation of different control strategies based on user’s preferences.

The results produced in the study indicated that optimization of control settings is a valid strategy for reduction of energy consumption of the existing HVAC systems.

References
